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Book Reviews

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Mathematical Theory of Elasticity

R. B. Hetnarski and J. Ignaczak, Taylor and Francis, New York, 2004, 821 pages, $125

Linear elasticity, along with hydrodynamics and analytical dynamics, may be regarded as model subjects in mechanics. Despite the idealized constitutive assumptions, mathematical solutions based on linear elasticity are widely used in engineering. Modern researchers in mechanics continue to seek inspiration and guidance from linear elasticity models as they explore untrodden paths in evolving areas.

Fundamental concepts in elasticity, including stress, strain, the conditions of equilibrium and compatibility, and variational principles, as well as a large body of exact solutions for various two-dimensional and three-dimensional domains, were developed in the past two centuries by mathematicians of the first rank. Their achievements show a level of analytical rigor, structural elegance, and generality of scope and application that has been attained in few other areas of physical science.

There is certainly no lack of good reference books on classical elasticity. Beginning with Love's pioneering treatise,[1] the subject has been well served by Timonshenko and Goodier,[2] Muskhelishvili,[3] Sokolnikoff,[4] Green and Zerna,[5] Gurtin,[6] and many others. New expositions of the subject continued to emerge in the past two decades, including those of Boresi and Chong,[7] Saada,[8] Barber,[9] and, most recently, Slaughter.[10]

Mathematical Theory of Elasticity, written by Hetnarski and Ignaczak and published in 2004, claims to be "the first new text on this subject for four decades." The list of new general theorems and applications in the book includes three-dimensional, compatibility-related variational principles of elastostatics, pure stress treatment of elastodynamics, and other results and solutions in elastodyamics and thermoelasticity.

The 13 chapters of the book may be grouped into three parts. The first chapter is a brief, refreshingly readable and balanced historical account of the development of elasticity entitled Creators of the Theory of Elasticity. Chapters 2–8 give clear expositions of fundamental concepts and balance laws, the mathematical formulation of linear thermoelasticity, variational principles in both static and dynamic elasticity, and various classical representations of general solutions in terms of scalar and vectorial potential functions. The third part, comprising the remaining five chapters, presents a body of analytical solutions of boundary and initial-value problems in isothermal and nonisothermal elastostatics and elastodynamics. The material is presented in a concise modern notation. Many important and interesting results are added as worked-out examples interspersed in the main text.

The second part of the book may serve as a sound, though not entirely adequate, introduction to linear elasticity for a beginning student. It covers many central topics in the theory but leaves out other important ones. For example, although the book contains many solutions for the circular cylindrical region, equilibrium conditions are given only in polar coordinates for the plane stress condition (p. 541) and without derivation. Strain-displacement relations in cylindrical coordinates (p. 502, Eq. 9.3.3) are quoted from Saada's book,[8] and integral compatibility conditions of multiply connected regions are not described, but referred to two books on thermal stresses in a footnote on page 178. As a result, the article on torsion of a prismatic bar in Chapter 9 makes no mention of rods with hollow cross sections.

More significant and intentional omissions include the skipping of the entire subject of complex variable theory in two-dimensional elasticity, of integral identities that lead to the boundary element approach, and approximate solution schemes based on various numerical methods. The third part of the book consists almost entirely of closed-form solutions of domains with elementary geometrical shapes such as the infinite and semi-infinite space and circular regions. Even infinite-series solutions based on separation of variables are excluded, though truncated series solutions are given in some examples of the Rayleigh–Ritz method. The reason for skipping the complex variable method was that it might be found in many standard textbooks. But this same reason is suspended when lengthy coverage is given to some special topics and solutions. On the other hand, important applications of elasticity in fracture and micromechanics using complex variable formulations and integral equation approaches are not to be found in the 800-plus pages of the book.

Despite the claim of "the most in-depth treatment of elasticity in years," the mathematical treatment given here is neither especially profound nor particularly novel. Many subjects have been covered in greater depth by Y.C. Fung in his thoughtful and useful book *Foundations of Solid Mechanics*,[11] and a thoroughly mathematical account of the elasticity theory in the framework of modern analysis and differential geometry is to be found in Marsden and Hughes.[12] The latter book also provides a glimpse of the current research on theoretical and mathematical elasticity documented mostly in the *Journal of Elasticity*.

If the book is to be examined not on the basis of what one expects in an up-to-date general introduction to classical elasticity, but with regard to its unique features and contents not easily accessible in other books, then one must be impressed by the special attention given to elastodynamics and thermoelasticity. The familiar Hamilton's principle is complemented by Gurtin's convolutional variational principles for initial-value problems, in different forms with and without counterparts in elastostatics. Furthermore, the chapters on two- and three-dimensional isothermal and nonisothermal elastostatic solutions in the third part of the book are matched by a more extensive coverage of dynamical solutions and elastic waves. Readers with a special interest in elastodynamics may find this book a valuable source of information. The selection of topics is often focused on the authors' research interest, such as the thermal stress induced by a fixed or time-dependent point heat source simulating laser heating. Furthermore, nearly all dynamical solutions presented here are limited to those with spherical or cylindrical (rotational) symmetry, besides the one-dimensional solutions in the last chapter.

The length of the book is due partly to explicit steps of mathematical manipulation that could be filled by most beginning students in mechanics, and occasionally to the authors' idiosyncratic ways of solution. The problems of Boussinesq and Cerruti for concentrated normal or tangential loads in a semi-infinite medium yield the fundamental solutions (Green's function) for the region, expressible in closed forms as algebraic functions of the coordinates. The authors used integral representations of the Dirac delta function in terms of Bessel functions or sinusoidal functions containing a variable parameter of integration [Eqs. (9.2.135) and (10.2.214)] and obtained Green's functions for both three- and two-dimensional regions (Secs. 9.2.3, 10.2.3, and 10.2.5) also as integrals in spectral representations, and subsequently converted them to closed form. But there is no compelling mathematical reason or intuitive appeal for using a spectral (Fourier) decomposition to obtain Green's function. Other authors obtained the solutions without taking this lengthy detour. The two-dimensional closed-form solutions are well known even in anisotropic elasticity (Ting[13]) and have been given geometrical interpretations as image singularities.

The inclusion of temperature effects in linear elasticity does not change the intrinsic mathematical structure and the totality of solutions. Indeed, linear thermoelasticity differs from elasticity not at all in the class of general solutions, but only in the particular solutions: those of thermoelasticity are associated with an effective body force combining thermal and mechanical loads. If the stress tensor S_{ij} is replaced by an effective thermomechanical stress $S_{ij}^* \equiv S_{ij} + \gamma T \delta_{ij}$, and if additionally the body force field b_i is replaced by an effective force $b_i^* \equiv b_i - \gamma T_{,i}$, where T is the temperature field and γ is the thermal expansion coefficient α multiplied by the elastic constant $3\lambda + 2\mu$, then the effective quantities satisfy the field equations $S_{ij,j}^* + b_i^* = 0$, $2\mu E_{ij} = S_{ij}^* - \lambda(3\lambda + 2\mu)^{-1}\ S_{kk}\delta_{ij}$, and the same compatibility equations remain valid for the strain tensor E_{ij}. That is, all field equations remain unchanged after replacement of the variables, and the particular solution of thermoelasticity is that associated with the effective body force field b_i^*. The mixed and traction boundary conditions are to be reformulated in terms of $S_{ij}^* n_j$ on a boundary surface with the unit normal vector n_j. All theorems and analytical results of linear elasticity may therefore be restated as corresponding theorems and results of thermoelasticity with a specified temperature field, and there is no need to reformulate a distinct theory for the linear thermoelastic problems, as it appears to be suggested in this and some other publications.

In summary, this attractively packaged book should find sympathetic readers with a keen interest in elastodynamics, particularly elastic waves and thermally induced pulses. The majority of students may find in the beginning chapters a clear and adequate introduction to elasticity and may learn substantially from its numerous worked examples. However, the exclusion of the complex variable method and various approximate schemes of solution may make it less desirable as a guide to many important mechanics problems and engineering applications. Whereas the exposition is sound and mathematically rigorous, the book does not live up to its claim as the most in-depth treatment of elasticity in years, in the sense that Ting's book[13] does in the area of anisotropic elasticity. The textbooks of Sokolnikoff[4] and Fung,[11] though nearly 50 and 40 years old, respectively, will continue to be useful, and so will be Gurtin's in-depth article in the *Encyclopedia of Physics*.[6] Perhaps the beauty of a well-developed mature subject such as linear elasticity is best glimpsed either in minimalist boutique tracts or in an up-to-date monumental work that remains to be written.

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